Applications of Exponential Functions

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GROWTH AND DECAY
COMPOUND INTEREST

Using exponential functions

$e = 2.71828$

- The exponential function $\boxed{f(x) = 1.26e^{0.247x}}$ models the gray wolf population of the Rocky Mountains, $f(x)$, x years after 1978. If the trend continues, project the gray wolf population in 2015.

$f(x) = 1.26e^{(.247)(37)}$

11,732 gray wolves

Growth and Decay Word Problems

- The equation $y = ab^t$ is used to represent a variety of growth and decay word problems—depending on the type of problem b will change

  y = ending amount

  a = initial amount

  t = time (number of time cycles)

  $\boxed{b = \text{base}}$

  - growth $(1 + r)$ (where r = rate)
  - decay $(1 - r)$
  - half-life $(.5)$
  - doubles, triples, etc. (2, 3, etc.)

growth $y = a(1+r)^t$

decay $y = a(1-r)^t$

$1 - .5$

Examples

- The population of Big Bang Town in the year 2000 was 2300 people. Assume that the population is increasing at a rate of $\boxed{2.25\%}$ per year. What will be the population in the year 2030? in 2050?

$y = a(1+r)^t$

$2.25\% = .0225$

$y = 2300(1+.0225)^{30}$

4,483

Examples

- The half-life of Howardium-234 is 25 days. If you start with 70 grams initially, how much is left after 100 days? after 400 days?

$$70(.5)^{\frac{100}{25}} = 70(.5)^4 = 4.375g$$

- When will there only be 10 grams remaining?

Compound Interest

- p=initial investment (Principle)
- r=interest rate (change to a decimal)
- n=# of times compounded in a year
  - Annually n=1
  - Monthly n=12
  - Quarterly n=4
  - Weekly n=52
  - Bimonthly n=24
- t=time in years

$$y = p\left(1+\left(\frac{r}{n}\right)\right)^{nt}$$

Compound Interest Examples

$y = P\left(1+\frac{r}{n}\right)^{nt}$

- Raj invests $500 into an account earning 7% annual interest compounded monthly. How much will he have after 15 years?

.07

n=12

$$y = 500\left(1+\frac{.07}{12}\right)^{(12)(15)}$$

$$y = \$1424.47$$

Compound Interest Examples

- Mrs. Wolowitz invests $10,000 into an account earning 3.6% annual interest compounded weekly. How much will she have after 28 months?

$\frac{28}{12} = \frac{7}{3}$

$10,000\left(1+\frac{.036}{52}\right)^{(52)(7/3)} = \$10,875.97$

- When will she have accrued $20,000?

$y = a(1+r)^t$ growth

$y = a(1-r)^t$ decay

Population

Compounded a specific # times/yr

$y = P\left(1+\frac{r}{n}\right)^{nt}$

Continuously Compounded Interest

- p=principal (initial investment)
- e=2.718
- r=interest rate decimal
- t=time

$$y = pe^{rt}$$

Compound Interest Examples

- Penny invests $23,000 into an account earning 6.2% annual interest compounded continuously. How much will she have after 6 years?

.062

$y = Pe^{rt}$

$23{,}000\,e^{(.062)(6)}$

$33,364.56

6.5

Compound Interest Examples

$y = Pe^{rt}$

- Leonard invests $1,000 at an annual interest rate of 5% compounded continuously. How much money will he have after 5 years?

$y = 1000e^{(.05)(5)}$

$1284.03

- ~~When will he have $3,000?~~

$y = a(1+r)^t$

$y = a(1-r)^t$

$y = P\left(1+\frac{r}{n}\right)^{nt}$

$y = Pe^{rt}$

Memorize